

Problems

Ted Eisenberg, Section Editor

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <<http://www.ssma.org/publications>>.

*Solutions to the problems stated in this issue should be posted before
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- **5373:** *Proposed by Kenneth Korbin, New York, NY*

Given the equation
$$\frac{2\sqrt{2}}{\sqrt{343 - 147\sqrt{5}} - \sqrt{315 - 135\sqrt{5}}} = \sqrt{x + y\sqrt{5}}.$$

Find positive integers x and y .

- **5374:** *Proposed by Roger Izard, Dallas TX*

In a certain triangle, three circles are tangent to the incircle, and all of these circles are tangent to two sides of the triangle. Derive a formula which gives the radius of the incircle in terms of the radii of these three circles.

- **5375*:** *Proposed by Kenneth Korbin, New York, NY*

Prove or disprove the following conjecture. Let k be the product of N different prime numbers each congruent to $1 \pmod{4}$. Let a be any positive integer.

Conjecture: The total number of different rectangles and trapezoids with integer length sides that can be inscribed in a circle with diameter k is exactly $\frac{5^N - 3^N}{2}$.

Editor's comment: The number for this problem carries with it an astrick. The astrick signifies that neither the proposer nor the editor are aware of a proof of this conjecture.

- **5376:** *Proposed by Arkady Alt, San Jose, CA*

Let $a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n$ be positive real numbers such that $a_1 < a_2 < \dots < a_n$ and $a_i \in (b_i, b_{i+1})$, $i = 1, 2, \dots, n - 1$. Let

$$F(x) = \frac{(x - b_1)(x - b_2) \dots (x - b_n)}{(x - a_1)(x - a_2) \dots (x - a_n)}.$$

Prove that $F'(x) < 0$ for any $x \in \text{Dom}(F)$.

- **5377:** *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*